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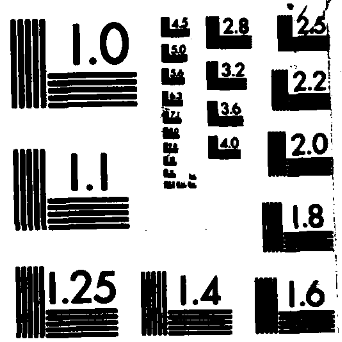

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PASSIVE SOURCE LOCALIZATION FROM SPATIALLY CORRELATED ANGLE-OF-ARRIVAL DATA

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PASSIVE SOURCE LOCALIZATION FROM SPATIALLY CORRELATED ANGLE-OF-ARRIVAL DATA

by

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Abstract

Bias and variance equations are presented for two-dimensional location estimators of a nonmoving point source of radiation in an isotropic, stationary random medium. The estimators are calculated from spatially correlated angle-of-arrival data which are collected simultaneously at two sensor positions and assumed to consist of true (unbiased) source angles plus zero-mean angular noise with equal variances at both sensors and negligible higher moments. Under these assumptions the square of the estimator bias is, in general, a quadratic function and the estimator variance a linear function of the spatial data correlation coefficient. However, for source ranges much larger than sensor separation, both the bias and the variance tend to increase linearly with decreasing correlation coefficient, whereas they tend to decrease with increasing sensor separation. The combined effect for a distant source in a stationary random medium, when evaluated for typical spatial wavefront autocorrelation functions, is a significant reduction in the estimator bias and variance dependence on sensor separation, as compared to the uncorrelated case. With minor modifications, the same results apply to the equivalent problem of using time-of-arrival data from three colinear sensor positions.

*Fellow, IEEE

Introduction

In sonar or radar, passive source localization is concerned with the estimation of an object's location from its emitted random radiation (acoustic or electromagnetic waves). For the two-dimensional case, this estimation of the source coordinates (x,y) requires simultaneous wavefront angle-of-arrival measurements by at least two separate sensors (or, equivalently, time-of-arrival measurements by at least three sensors). In many cases of practical interest, the angle measured by each sensor can be regarded as a random variable whose mean is the true (unbiased) source direction and whose variance is a constant (independent of source direction). In addition, the two random variables thus generated are jointly characterized by a spatial correlation coefficient which accounts for such factors as wavefront coherence, sensor assembly rigidity, etc. For sound propagating through the ocean, typical spatial coherence lengths range from 1 to over 10^3 meters.[1]

It is well known that even though the angles measured by the sensors may have unbiased means and constant variances, the source location estimated from these angles by direct calculation has a bias and variance which are functions of the source-sensor geometry. Several authors have investigated this problem under the assumption of spatially uncorrelated angle-of-arrival (or time-of-arrival) data.[2,3,4,5] In this paper, we present equations for the estimator bias and variance resulting from spatially correlated data (Section I). Section II examines the bias and variance dependences on correlation coefficient and source-sensor geometry and presents some of the results graphically. In Section III, the joint effects of spatial data correlation and sensor separation are examined for distant sources in a stationary random medium. Conclusions are drawn in Section IV.

I. General

In this section, we develop equations for the bias and variance of source location estimators calculated from correlated angle-of-arrival data. The two sensors are positioned at $x = \pm D$ in the x,y plane (Figure 1) and the true source location is (x_s, y_s) and (R,θ) in Cartesian and polar coordinates, respectively. The true source angles at sensors I and II are θ_1 and θ_2. All angles are measured counterclockwise from the positive x direction.

In terms of θ_1, θ_2, and D the true source location coordinates are as follows:

$$R = D\left(\frac{1+\cos^2(\theta_2-\theta_1)-2\cos(\theta_2-\theta_1)\cos(\theta_2+\theta_1)}{\sin^2(\theta_2-\theta_1)}\right)^{1/2} \tag{1}$$

$$\theta = \tan^{-1}\left(\frac{\cos(\theta_2-\theta_1)-\cos(\theta_2+\theta_1)}{\sin(\theta_2+\theta_1)}\right) \tag{2}$$

$$x_s = D\left(\frac{\sin(\theta_2+\theta_1)}{\sin(\theta_2-\theta_1)}\right) \tag{3}$$

$$y_s = D\left(\frac{\cos(\theta_2-\theta_1)-\cos(\theta_2+\theta_1)}{\sin(\theta_2-\theta_1)}\right) \tag{4}$$

(Note that these and the following equations are written in terms of $(\theta_2+\theta_1)$ and $(\theta_2-\theta_1)$ to simplify their interpretation for specific sensor-source orientations. Thus, for a "broadside" source located along the y-axis, $\theta_1+\theta_2 = \pi$.)

Now assume that the angle-of-arrival data generated at sensor positions I and II are random variables with means θ_1 and θ_2 (i.e., true source angles), variance σ^2 (same for both sensors and independent of direction), correlation coefficient ρ and negligible higher moments. The source coordinates (x,y) which are calculated from these data by (3) and (4) will then also

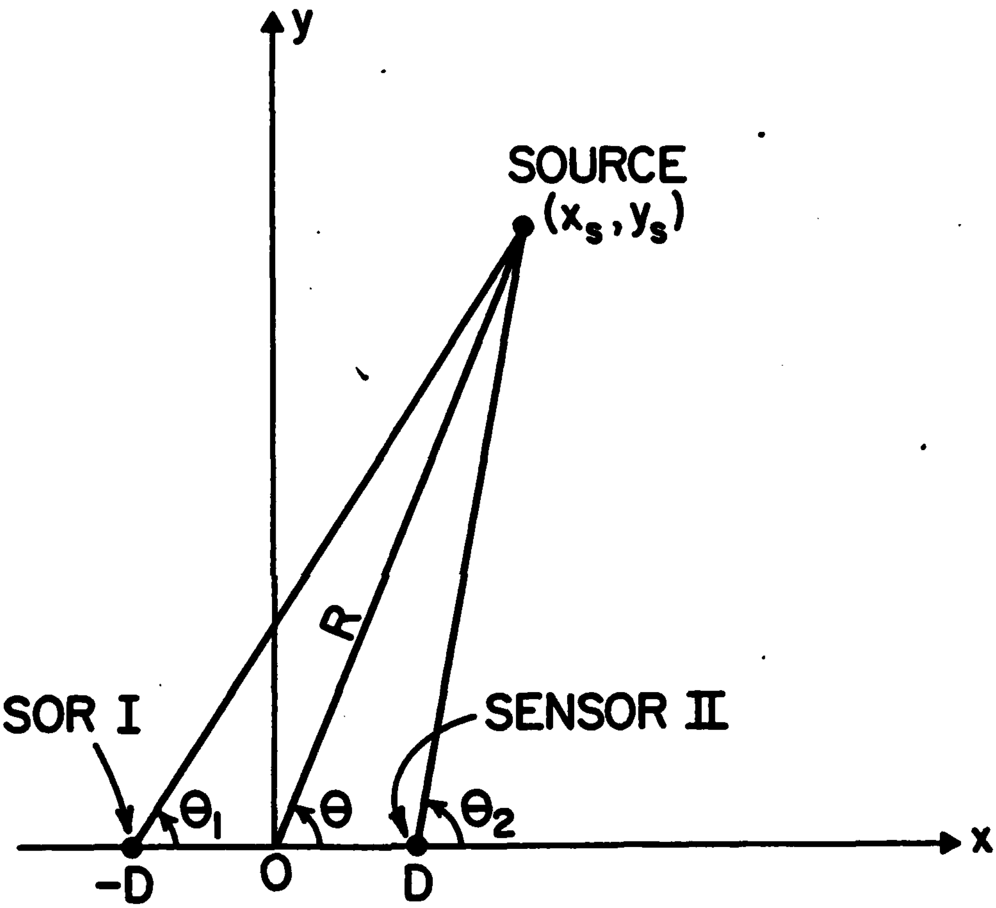

Figure 1. Source-Sensor Geometry.

be random variables whose moments can be determined in terms of the angle-of-arrival moments by expanding (x,y) into a Taylor series about the true source location (x_s, y_s).[6]

Thus, the mean and variance of x are given by

$$\overline{x} = x_s + \frac{\sigma^2}{2}\left(\frac{\partial^2 x_s}{\partial\theta_1^2} + \frac{\partial^2 x_s}{\partial\theta_2^2} + 2\rho\,\frac{\partial^2 x_s}{\partial\theta_1\partial\theta_2}\right)$$

$$\sigma_x^2 = \sigma^2\left(\left(\frac{\partial x_s}{\partial\theta_1}\right)^2 + \left(\frac{\partial x_s}{\partial\theta_2}\right)^2 + 2\rho\,\frac{\partial x_s}{\partial\theta_1}\,\frac{\partial x_s}{\partial\theta_2}\right)$$

where all terms involving moments higher than σ^2 and ρ have been neglected. Carrying out the indicated operations on (3) we obtain

$$\overline{x} = x_s + \frac{2D\sigma^2\sin(\theta_2+\theta_1)}{\sin^3(\theta_2-\theta_1)}\left(\cos^2(\theta_2-\theta_1) - \rho\right) \tag{5}$$

$$\sigma_x^2 = \frac{D^2\sigma^2}{\sin^4(\theta_2-\theta_1)}\Big(1 - \cos 2(\theta_2+\theta_1)\cos 2(\theta_2-\theta_1) + \rho[\cos 2(\theta_2+\theta_1) - \cos 2(\theta_2-\theta_1)]\Big) \tag{6}$$

Similarly, the mean and variance of y are

$$\overline{y} = y_s + \frac{2D\sigma^2\cos(\theta_2-\theta_1)}{\sin^3(\theta_2-\theta_1)}\left(1 - \cos(\theta_2+\theta_1)\cos(\theta_2-\theta_1) + \rho\left[\frac{\cos(\theta_2+\theta_1)}{\cos(\theta_2-\theta_1)} - 1\right]\right) \tag{7}$$

$$\sigma_y^2 = \frac{2D^2\sigma^2}{\sin^4(\theta_2-\theta_1)} \left([1 - \cos(\theta_2+\theta_1)\cos(\theta_2-\theta_1)]^2 + \sin^2(\theta_2+\theta_1)\sin^2(\theta_2-\theta_1) \right.$$

$$\left. - \rho[\cos(\theta_2-\theta_1) - \cos(\theta_2+\theta_1)]^2 \right) \quad (8)$$

The covariance of x and y is (again retaining only terms up to σ^2 and ρ):

$$C_{xy} = \sigma^2 \left(\frac{\partial x_s}{\partial\theta_1} \frac{\partial y_s}{\partial\theta_1} + \frac{\partial x_s}{\partial\theta_2} \frac{\partial y_s}{\partial\theta_2} + \rho \left[\frac{\partial x_s}{\partial\theta_1} \frac{\partial y_s}{\partial\theta_2} + \frac{\partial x_s}{\partial\theta_2} \frac{\partial y_s}{\partial\theta_1} \right] \right)$$

Performing the indicated operations on (3) and (4) yields

$$C_{xy} = \frac{2\sigma^2 D^2 \sin(\theta_2+\theta_1)}{\sin^4(\theta_2-\theta_1)} \left(\cos(\theta_2-\theta_1) - \cos(\theta_2+\theta_1)\cos2(\theta_2-\theta_1) \right.$$

$$\left. + \rho\left[\cos(\theta_2+\theta_1) - \cos(\theta_2-\theta_1)\right] \right) \quad (9)$$

It is convenient to consider the calculated source coordinates (x,y) as the rectangular components of a two-dimensional source-location estimator (i.e., a two-dimensional random vector) with mean $(\overline{x},\overline{y})$ and variance S. From (5) and (7), the magnitude B of the estimator bias (i.e., the Euclidean distance between the estimator mean $(\overline{x},\overline{y})$ and the true source location (x_s,y_s)) is obtained as

$$B = \left((\overline{x} - x_s)^2 + (\overline{y} - y_s)^2 \right)^{1/2} = A\left(E + \rho F + \rho^2 G \right)^{1/2} \quad (10)$$

where

$$A = \frac{2D\sigma^2}{\sin^3(\theta_2-\theta_1)}$$

$$E = \cos^2(\theta_2-\theta_1)\left(1 - 2\cos(\theta_2+\theta_1)\cos(\theta_2-\theta_1) + \cos^2(\theta_2-\theta_1)\right)$$

$$F = 2\cos(\theta_2-\theta_1)\left(\cos(\theta_2+\theta_1)(1 + \cos^2(\theta_2-\theta_1)) - 2\cos(\theta_2-\theta_1)\right)$$

$$G = 1 - 2\cos(\theta_2+\theta_1)\cos(\theta_2-\theta_1) + \cos^2(\theta_2-\theta_1)$$

The angle δ of the estimator bias is, from (5) and (7):

$$\delta = \tan^{-1}\left(\frac{\bar{y}-y_s}{\bar{x}-x_s}\right) = \tan^{-1}\left(\frac{(1-\rho)\cos(\theta_2-\theta_1)}{\sin(\theta_2+\theta_1)\left[\cos^2(\theta_2-\theta_1)-\rho\right]} - \cot(\theta_2+\theta_1)\right) \tag{10a}$$

Similarly, from (6) and (8), the estimator variance S (i.e., the mean-squared Euclidean distance between the estimator (x,y) and the estimator mean $(\bar{x},\bar{y})$) is obtained as

$$S = \sigma_x^2 + \sigma_y^2 = H[I + \rho J] \tag{11}$$

where

$$H = \frac{4D^2\sigma^2}{\sin^4(\theta_2-\theta_1)}$$

$$I = 1 - \cos(\theta_2+\theta_1)\cos(\theta_2-\theta_1)$$

$$J = \cos(\theta_2-\theta_1)\left(\cos(\theta_2+\theta_1) - \cos(\theta_2-\theta_1)\right)$$

II. Estimator Bias and Variance

The strong estimator dependence on the data correlation coefficient ρ is shown qualitatively in Figure 2 for a particular source-sensor geometry. The

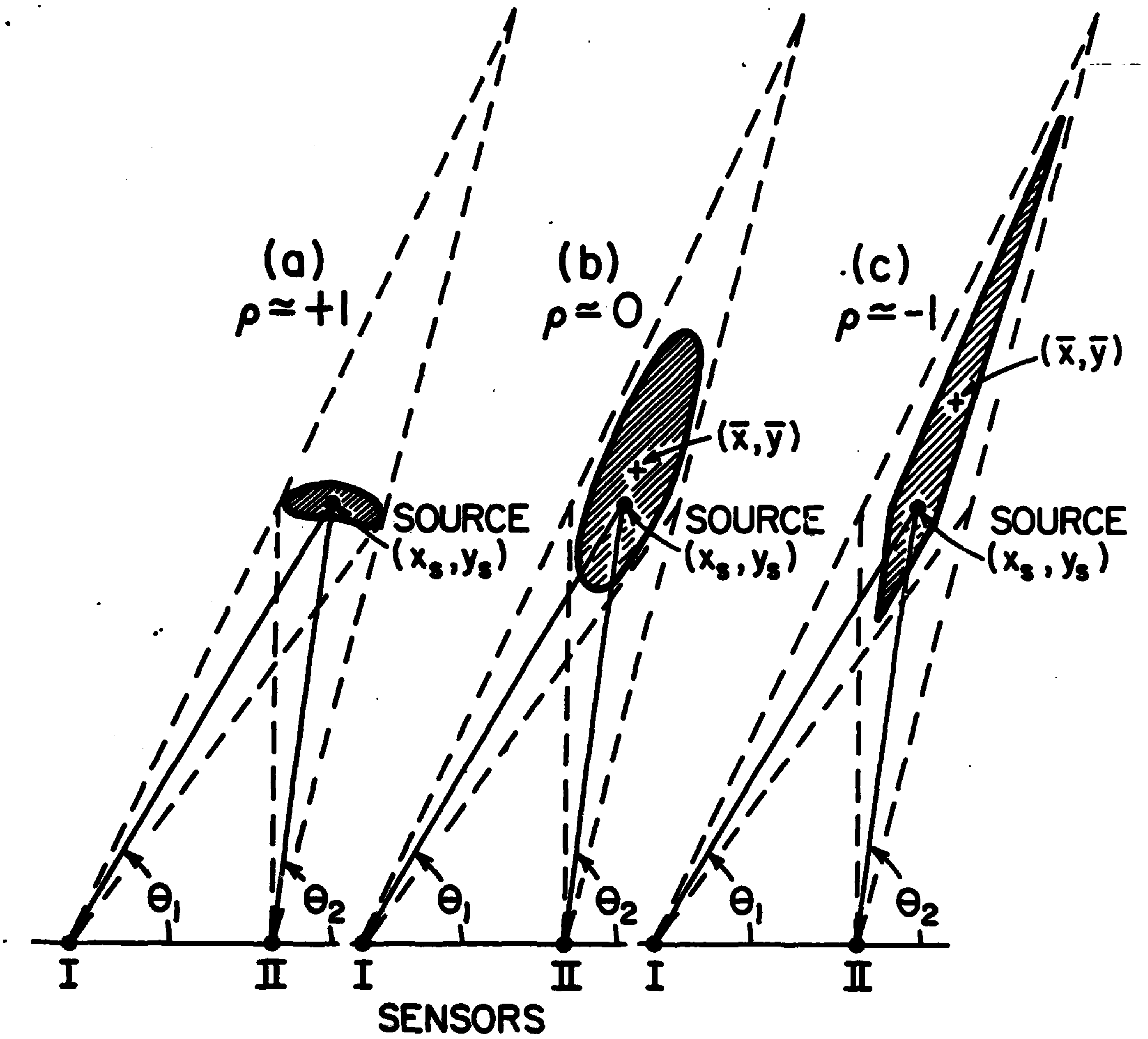

Figure 2. Source-Location Es' imator Di' :ributions (shaded) for (a) Positively Correlated Data (ρ ' 1), ') Uncorrelated Data (ρ = 0), and (c) Negatively Correlate' ,ata (ρ = -1).

cases $\rho \approx +1$ or $\rho \approx 0$ arise, for example, when the angle-of-arrival errors are primarily due to the random structure of the propagation medium between source and sensors, and the sensor separation is much smaller or much larger, respectively, than the medium's spatial coherence length. Similarly, the cases $\rho \approx +1$ or $\rho \approx -1$ can arise when the errors are primarily due to the random motion (rotational or translational) of the rigid two-sensor assembly relative to the propagation medium.

To investigate the estimator bias and variance dependences on ρ and on the source-sensor geometry in more detail, we make use of the inverse relations

$$\theta_2-\theta_1 = \tan^{-1}\left(\frac{\frac{2D}{R}\sin\theta}{1-\frac{D^2}{R^2}}\right) \tag{12}$$

$$\theta_2+\theta_1 = \tan^{-1}\left(\frac{\sin 2\theta}{\cos 2\theta - \frac{D^2}{R^2}}\right) \tag{13}$$

(Note that for source locations in the first quadrant, both $\theta_2-\theta_1$ and $\theta_2+\theta_1$ range over $(0,\pi)$). Substitution of (12) and (13) into (10) and (11) yields, after some straightforward but tedious algebra, the results for B, δ, and S:

$$B = W\left(L + \rho M + \rho^2 N\right)^{1/2} \tag{14}$$

where

$$W = \frac{R^3\sigma^2}{2D^2\sin^2\theta}\left(1 + \frac{D^4}{R^4} - \frac{2D^2}{R^2}\cos 2\theta\right)^{1/2}$$

$$L = \left(1 - \frac{D^2}{R^2}\right)^2$$

$$M = 2\left(1 - \frac{D^2}{R^2}\right)\left(\frac{D^2}{R^2}\cos 2\theta - 1\right)$$

$$N = 1 + \frac{D^4}{R^4} - \frac{2D^2}{R^2}\cos 2\theta$$

$$\delta = \tan^{-1}\left\{\left[1 + \frac{2D^2}{R^2}\left(1 - \frac{D^2}{R^2}\right)\left(L - \rho N\right)^{-1}\right]\tan\theta\right\} \tag{14a}$$

$$S = P\left(Q + \rho T\right) \tag{15}$$

where

$$P = \frac{R^4\sigma^2}{2D^2\sin^2\theta}\left(1 + \frac{D^4}{R^4} - \frac{2D^2}{R^2}\cos 2\theta\right)$$

$$Q = 1 + \frac{D^2}{R^2}$$

$$T = \frac{D^2}{R^2} - 1$$

Consider first the estimator bias B. Successive differentiation of (14) reveals that B is a minimum when $\rho = \rho_{min} = -\frac{M}{2N}$, or

$$\rho_{min} = \left(1 - \frac{D^2}{R^2}\right)\left(1 - \frac{D^2}{R^2}\cos 2\theta\right)\left(1 + \frac{D^4}{R^4} - \frac{2D^2}{R^2}\cos 2\theta\right)^{-1} \tag{16}$$

In Figure 3, ρ_{min} is plotted against the normalized source coordinates $(\frac{x_s}{D}, \frac{y_s}{D})$, where $(\frac{R}{D})^2 = (\frac{x_s}{D})^2 + (\frac{y_s}{D})^2$ from Figure 1. We note that $\rho_{min} > 0$ *for all source locations* <u>*except*</u> *those in the region* $\cos 2\theta < \frac{R^2}{D^2} < 1$.

By substituting (16) into (14), we obtain the <u>minimum</u> estimator bias $B_{min} = B_o\left(1 - \frac{M^2}{4LN}\right)^{1/2}$ or

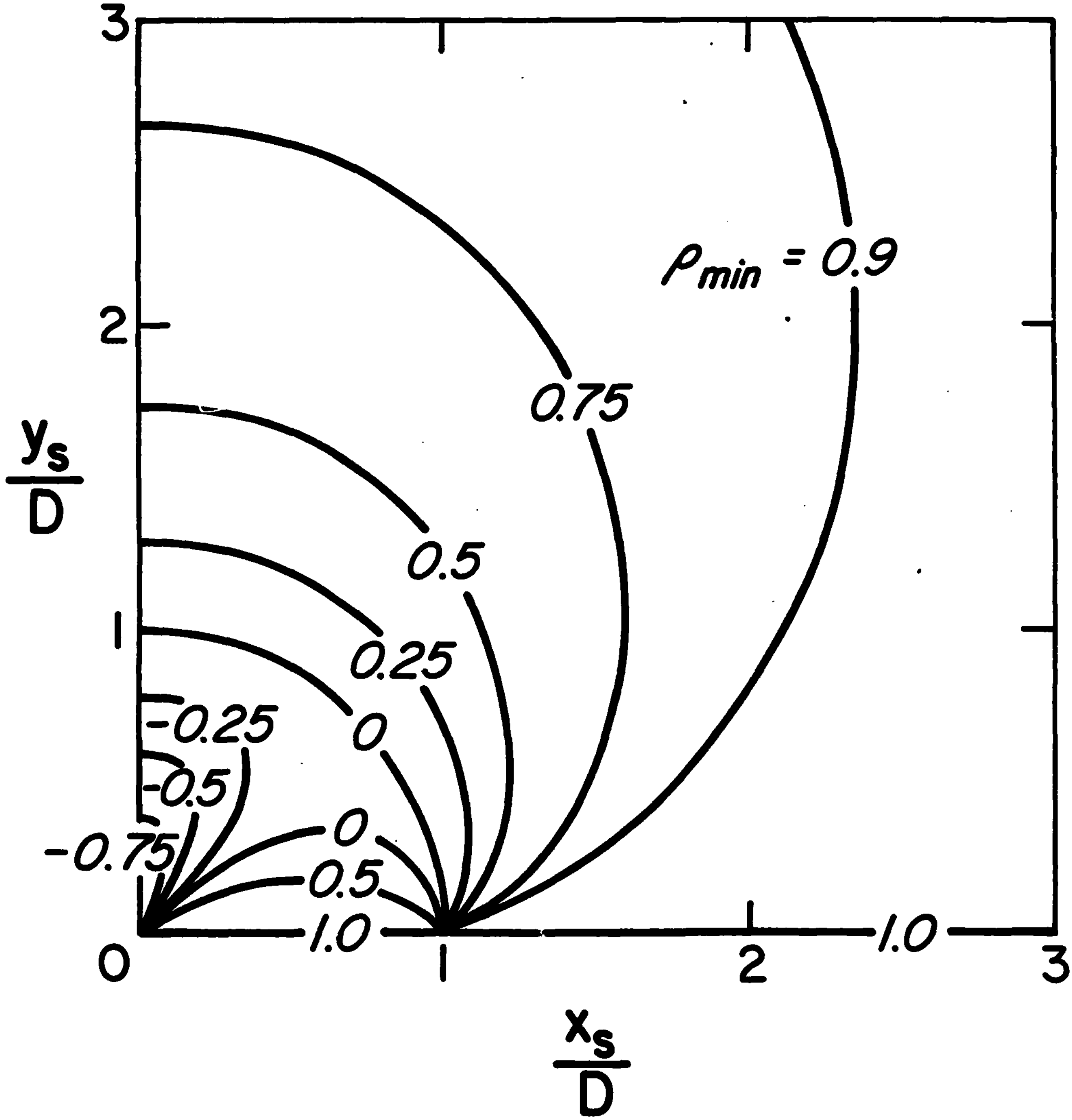

Figure 3. Data Correlation Coefficient (ρ_{min}) for Minimum Bias vs. Source Location.

$$\frac{B_{min}}{B_o} = \sin 2\theta \left(1 + \frac{R^4}{D^4} - \frac{2R^2}{D^2}\cos 2\theta\right)^{-1/2} \tag{17}$$

where $B_o = W[L]^{1/2}$ is the estimator bias obtained for uncorrelated data, i.e. for $\rho = 0$. In Figure 4, $\frac{B_{min}}{B_o}$ is plotted against $(\frac{x_s}{D}, \frac{y_s}{D})$.

The <u>maximum</u> estimator bias B_{max} is obtained by setting $\rho = +1$ or $\rho = -1$ in (14), according as to whether $\rho_{min} < 0$ or $\rho_{min} > 0$, respectively. The result is

$$\begin{aligned} \frac{B_{max}}{B_o} &= \Big(2(1 - \cos 2\theta)\Big)^{1/2}\left(1 - \frac{R^2}{D^2}\right)^{-1} \qquad ; \cos 2\theta < \frac{R^2}{D^2} < 1 \\ \frac{B_{max}}{B_o} &= \left(2 + \frac{2 + 2\frac{D^2}{R^2}\left(\frac{D^2}{R^2} - 2\right)\cos 2\theta}{\left(1 - \frac{D^2}{R^2}\right)^2}\right)^{1/2} \qquad ; \text{elsewhere} \end{aligned} \tag{18}$$

which is plotted in Figure 5 against $(\frac{x_s}{D}, \frac{y_s}{D})$.

Turning now to the estimator variance S, we note from (15) that S is a linearly <u>decreasing</u> function of ρ when $\frac{R^2}{D^2} > 1$ and a linearly <u>increasing</u> function of ρ when $\frac{R^2}{D^2} < 1$. Evaluating (15) for $\rho = -1$ and $\rho = +1$, we obtain the maximum and minimum variances, S_{max} and S_{min}:

$$\begin{aligned} \frac{S_{max}}{S_o} &= \frac{2}{1 + \frac{D^2}{R^2}}, \quad \frac{S_{min}}{S_o} = \frac{2}{1 + \frac{R^2}{D^2}} \quad ; \frac{R^2}{D^2} > 1 \\ \frac{S_{max}}{S_o} &= \frac{2}{1 + \frac{R^2}{D^2}}, \quad \frac{S_{min}}{S_o} = \frac{2}{1 + \frac{D^2}{R^2}} \quad ; \frac{R^2}{D^2} < 1 \end{aligned} \tag{19}$$

where $S_o = P \cdot Q$ is the estimator variance obtained for uncorrelated data, i.e. for $\rho = 0$. Thus, $\frac{S_{max}}{S_o}$ and $\frac{S_{min}}{S_o}$ are concentric circles in the $(\frac{x_s}{D}, \frac{y_s}{D})$ plane. Also, $\frac{S_{max}}{S_{min}} = \frac{R^2}{D^2}$ or $\frac{D^2}{R^2}$, according to whether the source location is outside or inside the circle $\frac{R^2}{D^2} = 1$, respectively.

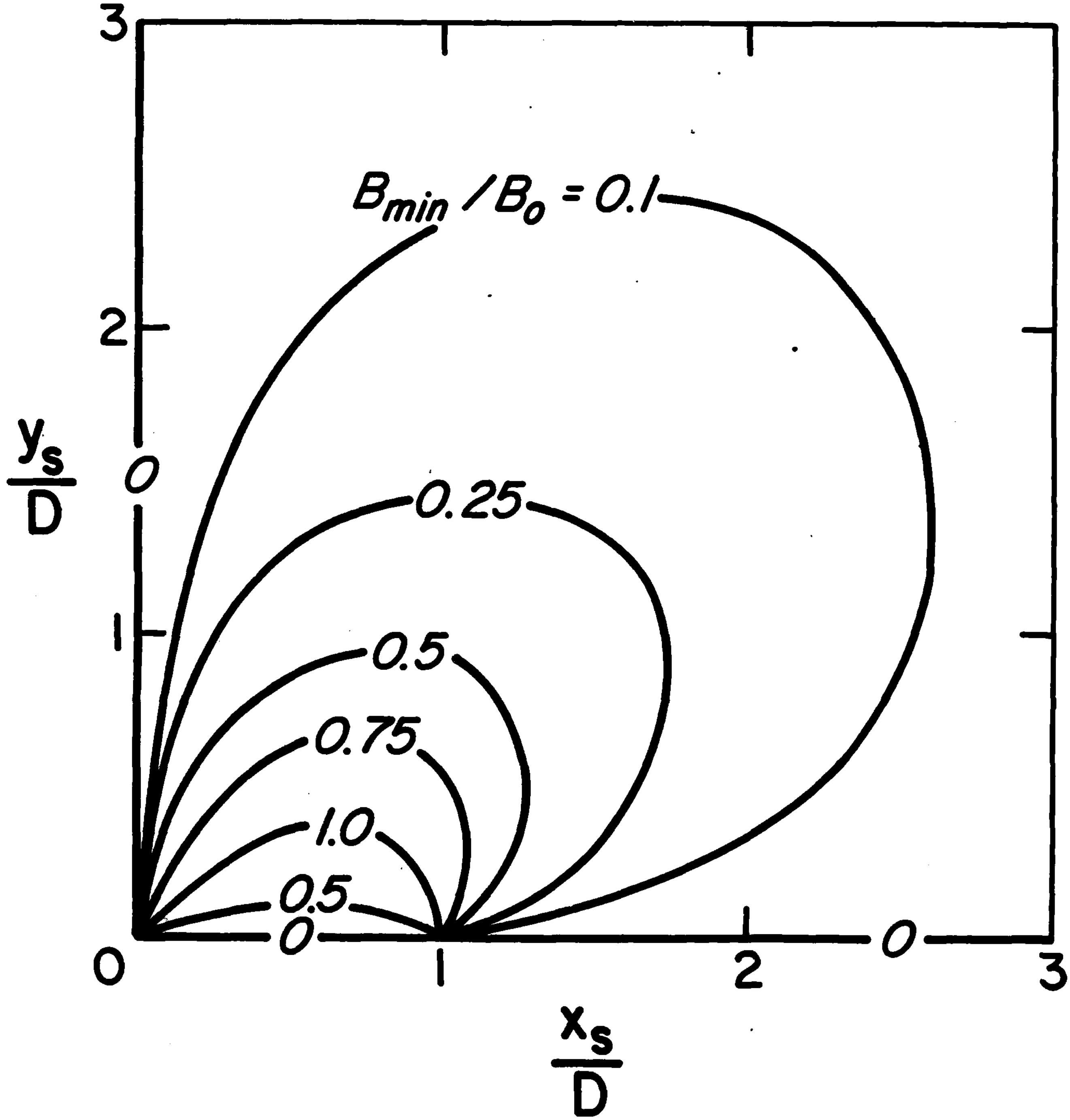

Figure 4. Normalized Minimum Bias (B_{min}/B_o) vs. Source Location.

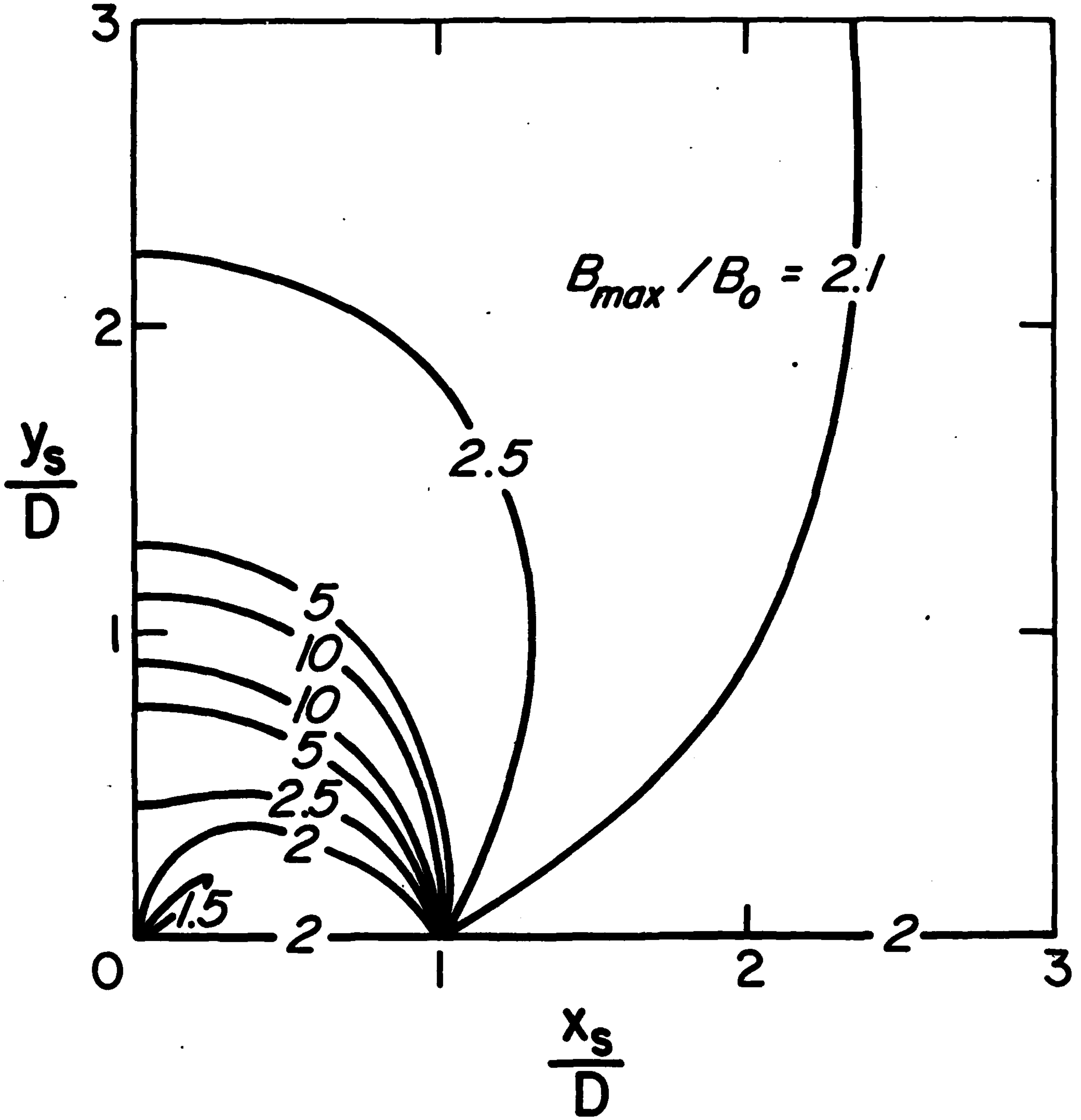

Figure 5. Normalized Maximum Bias (B_{max}/B_o) vs. Source Location.

Finally, it should be noted that since (14) to (19) are given in terms of the source-sensor parameters R, D, and θ (rather than the measured angles θ_1 and θ_2), these results apply equally well to the equivalent problem of using spatially correlated time-of-arrival measurements from three colinear sensors located at x = -D, 0, and +D (Figure 1). In such cases, the angular noise variance σ^2 used here must be evaluated from the corresponding time-delay variance which characterizes the difference in wavefront arrival times between adjacent sensor pairs. (An example is given in Section III.)

III. Effect of Sensor Separation on Distant Source Localization

For the case of a distant source (R >> D) we obtain from (14) and (15) the approximations

$$B \approx \frac{R^3\sigma^2}{2D^2\sin^2\theta}(1 - \rho) \tag{20}$$

$$S \approx \frac{R^4\sigma^2}{2D^2\sin^2\theta}(1 - \rho) \tag{21}$$

Inspection of (16) to (19) (or Figures 3 to 5) shows that in this case

$$\rho_{min} \approx 1, \frac{B_{min}}{B_0} \approx 0, \frac{B_{max}}{B_0} \approx 2, \frac{S_{min}}{S_0} \approx 0, \frac{S_{max}}{S_0} \approx 2 \quad .$$

In passing, we note that for the uncorrelated case (ρ = 0), (20) and (21) agree with the corresponding bias and variance expressions developed in reference 4, provided that the time-delay variance σ_τ^2 used there is converted into the corresponding angular variance σ^2 used here, according to

$$\sigma = \frac{2c\sigma_\tau}{D\sin\theta}$$

(where c = propagation velocity).

From (20) and (21) it is seen that for a fixed data correlation coefficient ρ both the estimator bias and variance increase with decreasing sensor separation D, whereas for a fixed sensor separation they both decrease with increasing correlation coefficient. These two effects therefore counteract each other in cases where ρ is a decreasing function of D. Such situations arise, for example, when the angle-of-arrival errors are primarily due to the random structure of the propagation medium between source and sensors.

To gain some insight into this tradeoff between ρ and D, we represent both (20) and (21) by the functional relation

$$f(D,\rho) = \frac{K}{D^2}(1 - \rho) \tag{22}$$

where K is independent of D and ρ. The net effect of a small change in sensor separation is therefore given by the derivative

$$\frac{df(D,\rho)}{dD} = -\frac{2K}{D^3}\left(1 - \rho + \frac{D}{2}\frac{d\rho}{dD}\right) \tag{23}$$

For the special case where the data are uncorrelated for all sensor separations we have $\rho = 0$, $\frac{d\rho}{dD} = 0$, and

$$\left[\frac{df(D,\rho)}{dD}\right]_0 = -\frac{2K}{D^3} \tag{24}$$

For the more general case we treat the angle-of-arrival data as a stationary random process with spatial autocorrelation coefficient $\rho(D)$, where $\rho(0) = 1$, $\rho(\infty) = 0$, and $\frac{d\rho}{dD}$ is restricted by the requirement that $\rho(D)$ must be positive-definite (i.e., its Fourier transform must be nonnegative for all spatial wavelengths). Since this restriction cannot be expressed in a convenient algebraic form, we evaluated (23) for some typical spatial autocorrelation coefficients and different values of $\rho(D)$. In Table I, the results are normalized relative to the uncorrelated case (24). The fact that $\frac{df(D,\rho)}{dD} < 0$ for all cases considered suggests that, in practice, an increase in sensor separation will always result in a net decrease of estimator bias and variance for distant targets. However, it is seen from Table I that the rate of this decrease may be much smaller for the case of spatially correlated data than for uncorrelated data.

IV. Conclusions

We have investigated the effect of spatial correlation between two-sensor angle-of-arrival data on the source location estimates calculated from these data. The conclusions are:

1. Both the estimator bias and variance are strong functions of data correlation and source-sensor geometry.
2. For distant sources, estimator bias and variance both tend to decrease linearly with increasing correlation coefficient.
3. Data correlation dependence on sensor separation, as given by typical spatial autocorrelation coefficients in stationary random media, can cause greatly reduced estimator bias and variance dependences on sensor separation.

4. The general estimator bias and variance equations (10,11,14,15) are limited in accuracy by the assumption that the angle-of-arrival data distributions have negligible moments above the second.

V. Acknowledgements

Helpful comments by H.D. Brunk and S.I. Chou are gratefully acknowledged. This work was supported under ONR Contract N00014-81-K-0814 and NUWES Contract N00406-82-D-0675.

$\rho(D)$	$\frac{df}{dD} \Big/ \left(-\frac{2K}{D^3}\right)$			
	(1)	(2)	(3)	(4)
0.9	0.002	0.005	0.050	0.053
0.8	0.013	0.021	0.10	0.11
0.7	0.030	0.050	0.15	0.18
0.6	0.055	0.094	0.20	0.25
0.5	0.091	0.15	0.25	0.33
0.4	0.14	0.23	0.30	0.42
0.3	0.20	0.34	0.35	0.52
0.2	0.27	0.48	0.40	0.64
0.1	0.31	0.67	0.45	0.78

Table I: Estimator Bias and Variance Dependence on Sensor Separation for the Following Autocorrelation Coefficients ($a > 0$):

(1) $\rho(D) = \dfrac{\sin(aD)}{aD}$

(2) $\rho(D) = \exp(-a^2D^2)$

(3) $\rho(D) = \begin{cases} 1-a|D| & ; \ |D| < \frac{1}{a} \\ 0 & ; \ |D| > \frac{1}{a} \end{cases}$

(4) $\rho(D) = \exp(-a|D|)$

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